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Optimization Wavelet Thresholding in Non-Stationary Time-Series Analysis for Treatments Tuberculosis Case Patients

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Abstract:

Non-stationary time series (TS) analysis has gained great interest over the last few decades in various applied sciences. In fact, several decomposition methods were developed to extract various components (e.g., seasonal, trend, and sudden components) from non-stationary TS, which allows a better interpretation of temporal variability. Wavelet Thresholding (WT) has been successfully applied over a tremendous range of fields to decompose non-stationary TS into the time-frequency domain. There are two types of wavelet estimators, namely linear wavelet estimators and nonlinear wavelet estimators. Linear wavelet estimators can be analyzed using the Multiresolution Analysis (MRA) approach, while nonlinear wavelet estimators are called Wavelet Thresholding (WT). Wavelet Thresholding emphasizes wavelet reconstruction using the largest number of coefficients or you could say only coefficients that are greater than the value taken, while other coefficients are ignored. There are various challenges for optimization related to wavelet transform, such as selecting the type of wavelet, selecting an adequate parent wavelet, selecting the scale, combining wavelet transform and machine learning algorithms. Apart from that, there are several factors that influence the smoothness of the estimation, namely the type of wavelet function, type of threshold function, threshold parameters, and level of resolution. Therefore, in this paper the optimal threshold value will be obtained in analyzing the data. The Wavelet Thresholding method provides a smaller MSE, MAPE, SNR and Energy value compared to the wavelet method with the Multiresolution Analysis (MRA) approach. In this case study, Wavelet Thresholding is considered better in time series data analysis.

Keywords : Wavelet Thresholding Estimator, Multiresolution Analysis, Tuning Parameter, Non stationary

Introduction

Tuberculosis (TB) continues to be a major global health concern. 10.6 million cases of tuberculosis were predicted to have been identified globally in 2021, an increase of about 600,000 cases above the 10 million cases reported in 2020. 4.27 million (39.7%) of these 10.6 million instances have not yet been found and reported, whereas 6.4 million (60.3%) of the cases have been recorded and are undergoing treatment. Around 30,000 people get TB each day, and approximately 4,400 people die from the disease, according to a WHO report from 2023 (*Global tuberculosis report 2023*, 2023).

Indonesia had the third-highest number of tuberculosis cases worldwide in 2021, but by 2022 it had risen to second

position, behind China and ahead of India. An estimated 969,000 TB cases, or one case every 33 seconds, were reported in Indonesia in 2022, a 17% rise from the 824,000 cases reported in 2021. In Indonesia, there are 354 cases of tuberculosis (TB) for every 100,000 individuals, or 354 victims for every 100,000 Indonesians. The aim of eradicating tuberculosis by 2030 is seriously hampered by this circumstance. Information technology will be used to support the provision of comprehensive TB prevention and control services, including risk factor analysis and the development of TB incidence prediction models, in order to meet this goal (Saifullah et al., 2021).

A reliable prediction model for TB incidence treatment must be developed because the number of TB treatment

occurrences vary greatly (Wang et al., 2023). Several hybrid approaches are used to predict the fluctuating patterns in time-series data (noise) in order to achieve reliable forecasts. An effective denoising technique for predicting systems using non-stationary data is signal decomposition. The prediction model's performance can be improved by employing signal decomposition to separate the non-stationary components of the time-series signal (Cao et al., 2013).

The application of wavelets for forecasting time series data, particularly fluctuating data, is rapidly advancing. This is because Discrete Wavelet Transformation (DWT) is considered particularly suitable for time series data because it produces wavelet coefficients and scales that correspond to the length of the data at each level of decomposition. The drawbacks of DWT filtering, which can be applied to any sample size, are mitigated by this feature. Dalam konteks optimization algorithm, this paper proposes the combination of numerical parameters and categorical/structural parameters, such as method selection. This method includes wavelet adaptive thresholding optimization at each level of decomposition by examining all possible combinations of numerical parameter values and available thresholding methods in the parameter space.

One of the important elements in signal denoising is wavelet threshold optimization. Many methods are proposed to improve performance. Artificial Fish Swarm Algorithm (AFSA) has been used to optimize wavelet thresholds; conventional methods show worse results (Mingyan and Dongfeng, 2005). To select the threshold, recent research has seen that optimization algorithms such as Aquila Optimizer (AO), Gradient-Based Optimizer (GBO), and Modified Gray Wolf Optimizer (GNHGWO) have good potential for benchmark signal denoising (Hu et al., 2021). According to Zhu Li et al. (Zhu et al., 2023), edge detection-based optimization can maintain edge information while removing noise. This can result in a better signal-to-noise comparison. Additionally, wavelet coefficient thresholds for grayscale images corrupted by additive white Gaussian noise have been discovered by using entropy-based optimization, which outperforms universal soft.

The selection of threshold parameters in wavelet thresholding, a critical approach in nonparametric regression estimation, greatly affects the estimated function's smoothness (Othman, 2020). To achieve optimal function estimation, it is necessary to identify the ideal threshold value, which can be done in a number of ways. These include multiple hypothesis testing and the False Discovery Rate (FDR) process, where the threshold value and ensuing function smoothness are influenced by the significance level (Hassan et al., 2022). Efficient multilevel thresholding methods in image processing have been developed by Figueroa et al (Figueroa-López and Mancini, 2019). These algorithms analyze image histograms in order to minimize or maximize objective functions. Block thresholding estimators for wavelet regression have been researched, taking into account the impacts of block size on global.

Literature Review

Wavelet Analysis

A popular mathematical method for signal and image analysis is wavelet analysis. The wavelet transform's widespread appeal in recent years can be attributed to its capacity to accurately characterize nonstationary phenomena. In contrast to the Fourier transform, wavelets are used far more frequently and are receiving a lot of attention. This is mostly explained by wavelets' remarkable capacity to estimate smooth functions and analyze a wide range of data types, including nonstationary and stationary data. On the other hand, there are certain limits with the Fourier transform as an analytical tool, particularly when working with nonstationary data. The inability to localize the time domain and the comparatively higher computing complexity of decomposition techniques are two examples of these restrictions.

For a long time, signal estimation has been heavily reliant on the study of noise reduction in many different scientific fields. Recent studies have shown that noise hinders the efficacy of a number of techniques, such as identification, parameter estimation, and prediction accuracy (Liu, 2020). Thus, early data manipulation is strongly preferred in order to reduce noise interference without sacrificing the intrinsic dynamics of the underlying signal (Bing et al., 2020).

Discrete Wavelet Decomposition (DWD).

A pre-processing technique called DWD makes it easier to project time series onto a group of orthonormal basis functions. The purpose of this specific transformation is to glean more information from the initial time domain data. Decoding the data at different frequencies can be done to do signal analysis after DWD has been applied to it. Low-frequency components typically exhibit discernible patterns generated from the original data, which helps with the estimate process, whereas high-frequency components have the potential to introduce noise. In this study, weekly Henry Hub spot prices are broken down into four separate subseries using DWD.

Wavelet Transformation in Time Series Forecasting

Because wavelet transformation can handle non-stationary data, it has grown in popularity in time series forecasting. The use of Discrete Wavelet Transformation (DWT) in time series analysis is especially noteworthy. DWT can be applied to any sample size since its decomposition technique yields wavelet coefficients and scales at each level that match the length of the data (Ilu and Prasad, 2024). This flexibility is essential for examining TB incidence statistics that fluctuate since they frequently show non-stationary features.

Signal Decomposition and Denoising

A key component of forecasting method optimization for non-stationary data is efficient signal decomposition. Through the use of denoising techniques, prediction models are able to isolate non-stationary components and achieve drastically improved accuracy. According to Elshekhdri, Mohamedamien, and Ahmed (Elshekhdri, Mohamedamien and Ahmed, 2023). DWT has been especially successful in signal decomposition, offering a stable framework for denoising time series data.

Wavelet Thresholding Techniques

Wavelet thresholding, which involves choosing a threshold value to filter out noise while keeping important signal components, is an essential method for denoising time series data. Numerous thresholding techniques, such as hard and soft thresholding, have been investigated; based on the properties of the data, each technique offers unique benefits (Vimalajeewa *et al.*, 2023). Effective denoising depends on the thresholding method selected and the ideal threshold values determined.

Hyperparameter Optimization in Wavelet Thresholding

Optimizing hyperparameters is essential to improving wavelet thresholding performance. The effectiveness of the denoising process is directly influenced by hyperparameters like the threshold values and the decomposition level. Finding the best configurations for complicated datasets has showed promise when using exhaustive search techniques, which examine every possible combination of hyperparameter values (Chowdhury *et al.*, 2022). This thorough process guarantees the optimal settings are chosen, improving forecasting accuracy and denoising.

Combining Numerical and Categorical Parameters

The integration of numerical and categorical parameters in wavelet thresholding optimization has been the focus of recent developments. This all-inclusive strategy, which combines method selection with numerical parameter tweaking, enables a comprehensive parameter space investigation. These combinations improve the overall performance of the model by facilitating wavelet adaptive thresholding optimization that is customized for each level of decomposition (Balogun *et al.*, 2021).

Research Methods

In this particular research, research was conducted to determine the use of segmentation-based wavelet threshold optimization. Several stages are carried out, namely preprocessing, segmentation, threshold optimization and signal reconstruction to produce optimal denoising. At the threshold optimization stage, it is carried out on each

segment that is formed, using k-means, the aim is to find clusters that have the same pattern or characteristics, making it easier to treat with the right method for each segment. The decomposition function is evaluated using a different wavelet basis for each segment. Threshold optimization is carried out by optimizing the estimator and optimizing the hyperparameter combination. In the estimator optimization, a combination of ridge regression and lasso regression algorithms is used, with regularization complexity control. Meanwhile, at the hyper parameter optimization stage, a Bayesian optimization algorithm is used to control the complexity of timeseries cross validation. To find the appropriate threshold value for each segment, a composite evaluation of Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Signal-to-Noise Ratio (SNR) and Energy is used. After finding the optimal threshold for each segment at each cD level, then. The approximate wavelet coefficients and wavelet coefficients at each cD level are reconstructed using the inverse wavelet transform ISWT (Inverse Stationary Wavelet Transform). So the signal returns to its original form cleanly and without noise denoising).

The level of smoothness of the curve in the Function Approach using the wavelet thresholding method is influenced by various parameters. These parameters include the selected wavelet function type, resolution level, thresholding function type, and thresholding value (IEEE Communications Society and Institute of Electrical and Electronics Engineers). To obtain optimal results, optimization is carried out for each parameter. The optimization process consists of two parts. The first part focuses on optimizing the wavelet basis and resolution levels used in the wavelet transform process, as well as the threshold function used in the thresholding process (Bayer, Kozakevicius and Cintra, 2019). Next, the second part involves optimizing specific threshold values that serve as threshold limits in the thresholding process. This particular optimization is achieved through the utilization of the bayesian optimization method.

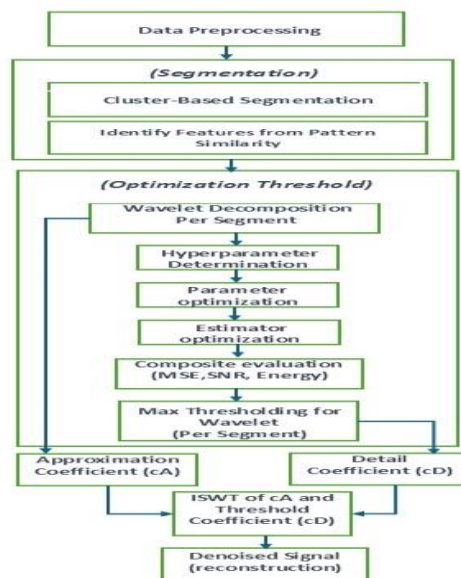


Figure 1. Overall system model of the present study

Result and Discussion

Optimization Approach with Bayesian Optimization.

Functions to find optimal thresholds efficiently by setting various adaptive thresholding parameters such as wavelet type, level, and thresholding methods identified in hyperparameter space. This Bayesian optimization algorithm is a method or technique used to find the best parameters for the estimator based on a predetermined objective function, namely Minimization of Mean Squared Error (MSE), Minimization of Root Mean Squared Error (RMSE), Maximization of Signal-to-Noise Ratio (SNR) , Energy Preservation (seeing signal energy after denoising). The steps include various strategies to explore the parameter space and refine the parameters to achieve optimal results. This optimization algorithm will evaluate various parameter combinations by training an estimator and calculating performance based on evaluation metrics. Bayesian Optimization will try different parameter combinations and update the search strategy based on previous results. Bayesian Optimization to optimize the regression model

used in this research.

Wavelet Decomposition Results

Figure 2. Shows the components resulting from the discrete wavelet decomposition process at level 3. Starting from the bottom up, we observe the approximation components and detailed components with the approach decomposed up to level three. The approximation and detail components are presented in three sequences. This discrete wavelet decomposition can be achieved through various approaches such as Daubechies, Coiflets, Symlets, or Discrete Meyer. Among these wavelets, Daubechies and Symlet allow perfect reconstruction with the maximum number of missing moments. Symlets show perfect symmetry, whereas Daubechies do not. Due to the fact that symmetry can limit flexibility in representing data, Daubechies wavelets are chosen. Three optimal mean square errors (MSE) are used to determine the total number of missing moments of the Daubechies wavelet within the available perfect reconstruction range.

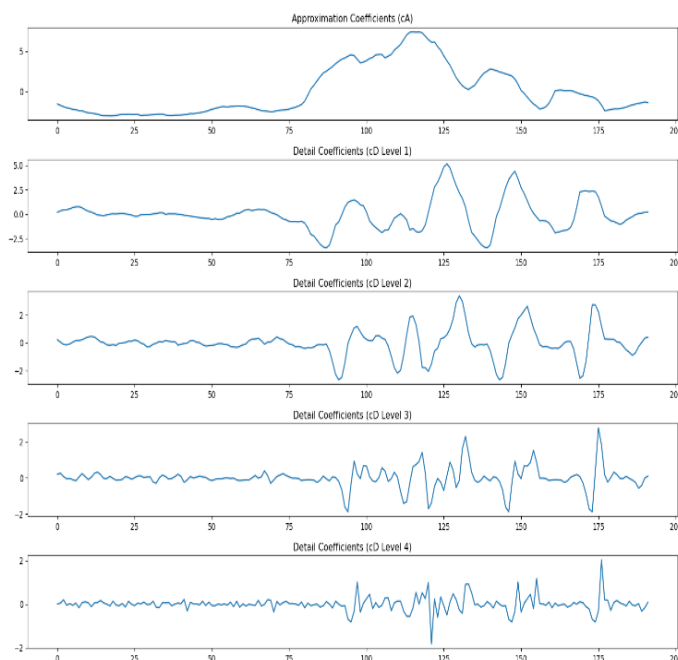


Figure 2. Level 4 Coefficient Decomposition

Adaptive Threshold Optimization at Each cD Level

To carry out specific threshold optimization at each cD level (adaptive threshold) in wavelet denoising, it is necessary to design an approach that can find the optimal threshold value for each cD level independently. Steps for Adaptive Threshold Optimization at each cD Level, Figure 1. is a

signal decomposition using wavelet transform. In the wavelet transform stage, the original signal is carried out to obtain the approximation coefficient (cA) and detail coefficient (cD) at various levels. The decomposition results using db4 with 4 levels, you will get one cA (cA4) and four cD (cD1, cD2, cD3, cD4). Initialization parameters for optimization are as in table 1.

Table 1. Initialization Parameters for Optimization.

Hyperparameter Space	Value
Wavelet_methode	'db3', 'db4', 'haar', 'sym4'
Max_level	1, 2, 3, 4, 6
Threshold_cD	'hard', 'soft'
Thresholding_method_cD	'sureshrink', 'statistical'
Sigma_method	'median', 'mean'
Denoising_method	'wavelet', 'kalman', 'lowpass'
Transform_type	'wavedec', 'swt'

For each cD level (eg cD1, cD2, cD3, cD4), threshold optimization must be carried out independently. This involves finding the optimal threshold value for each level that minimizes error or meets other predetermined criteria specified as evaluation metrics, such as m Minimization of Mean Squared Error (MSE), Minimization of Root Mean

Squared Error (RMSE), Maximization of Signal-to-Noise Ratio (SNR), Energy Preservation (seeing signal energy after denoising). The evaluation results produced optimal threshold values and the best parameters were found at each CD level as in Table 2.

Table 2. Best threshold and parameter optimization results vs wavelet threshold with MRA optimization.

Level cD	with threshold adaptive optimization.	with MRA optimization
Level 1	'[0.3453906651221019, 'hard', 'statistical', 'median', 'db4', 4, 'wavedec', 'kalman'] Score: 2.6013922343357636	'[0.01, 'hard', 'statistical', 'median', 'db4', 4, 'wavedec', 'kalman'] Score: 2.8013922 343357636
Level 2	[0.43141738585649325, 'soft', 'sureshrink', 'mean', 'db4', 3, 'swt', 'lowpass'] Score: 2.6220601669466214	'[0.01, 'hard', 'statistical', 'median', 'db4', 4, 'wavedec', 'kalman'] Score: 2.8013922343357636
Level 3	[0.7485693892533252, 'soft', 'statistical', 'mean', 'db3', 2, 'wavedec', 'lowpass'] Score: 2.6459523661268903	'[0.01, 'hard', 'statistical', 'median', 'db4', 4, 'wavedec', 'kalman'] Score: 2.8013922343 357636
Level 4	[0.2948953189819348, 'hard', 'sureshrink', 'median', 'db3', 2, 'swt', 'kalman'] Score: 2.6237769439631323	'[0.01, 'hard', 'statistical', 'median', 'db4', 4, 'wavedec', 'kalman'] Score:2.8013922343357636

Table 2. shows the combination of various parameters used for thresholding optimization at each cD level (detail coefficient) in wavelet denoising. Each element in the list represents an optimized value for a particular parameter with a score resulting from various evaluation metrics. This score value is the result of model evaluation after thresholding with a combination of these parameters. This value is based on evaluation metrics such as MSE, RMSE, SNR, MAPE

and Energy. The lower the score value, the better the model performance on test or validation data. The resulting score shows that thresholding optimization is better than thresholding with MRA optimization. The threshold value in thresholding optimization is also more specific than thresholding with MRA optimization, which tends to be uniform in value without considering the signal character conditions at each cD level.

Table 3. Optimization results for each cD level (cD1, cD2, cD3, cD4) based on evaluation metrics.

Level cD	Evaluations Metrics with threshold adaptive optimization	Evaluations Metrics with MRA optimization
Level 1	'MSE': 4.213170188756314, 'RMSE': 2.052600835222551, 'SNR': -6.246090025744886, 'Energy': 4.175551678764966, 'MAPE': 423.13590325929766	MSE': 0.11913832975470, 'RMSE': 0.34516420694317, 'SNR': 9.239484 92733044, 'Energy': 0.72435302 4475 08, 'MAPE': 55.49758961853511
Level 2	'MSE': 4.2108737290600535, 'RMSE': 2.052041356566688, 'SNR'-6.243722184816147, 'Energy': 4.179670453100956, 'MAPE': 425.3983814938626	MSE': 0.01001412712095, 'RMSE': 0.10007061067542, 'SNR': 19.99386 89989754, 'Energy':0.91783639813059, 'MAPE': 20.5765735770563
Level 3	'MSE':4.197052914615439, 'RMSE': 2.0486710118062974, 'SNR':-6.229444446417098, 'Energy':4.198374843980905, 'MAPE': 417.49132655901417	MSE': 0.00434237946719, 'RMSE': 0.06589673335754, 'SNR': 23.62 27 22275632366, 'Energy': 0.96249080 395291, 'MAPE': 11.92048229550 61
Level 4	'MSE': 4.1609388616476295, 'RMSE': 2.0398379498498476, 'SNR': -6.191913345758994, 'Energy': 4.205626116429856, 'MAPE': 419.76682953927514	MSE': 0.04515428357690, 'RMSE': 0.21249537307176, 'SNR': 13.4530104389819, 'Energy':0.80606529220261, 'MAPE': 37.07449927049839
Combination (1, 2, 3, 4)	'MSE': 5.284582361690713, 'RMSE': 0.000726951329986, 'SNR': 62.76989329109645, 'Energy': 0.999998173432878, 'MAPE': 0.074816357408244	MSE': 0.00058551338749, 'RMSE': 0.02419738389783, 'SNR':32.324631705309564, 'Energy':0.99833103814587, 'MAPE': 4.30960225778361

For each cD level (cD1, cD2, cD3, cD4) threshold optimization is carried out independently. This involves

finding the optimal threshold value for each level that minimizes error or meets other predetermined criteria specified as an evaluation metric. Table 3 shows the results

of calculating evaluation metrics at each cD level and then used as a reconstruction by combining all cD levels where the results show that threshold optimization with Bayesian optimization is better than without optimization.

Signal Reconstruction.

After finding the optimal threshold value for each cD level, the denoising process is then carried out on the detailed coefficients to be used as a signal reconstruction process. Signal reconstruction uses inverse wavelet transform (ISWT or IDWT) to reconstruct the signal from denoised

coefficients. This reconstruction involves recombining the approximation coefficients (cA) and detail coefficients (cD) that have been denoised at each level. After the signal is reconstructed, evaluate the results using relevant evaluation metrics such as MSE, MAPE, SNR and Energy so as to produce a clean reconstructed signal free from noise. Figure 2. below is a reconstruction of wavelet decomposition without parameter optimization, while Figure 3. is the reconstruction result of a combination of Threshold Optimization carried out for each cD level.

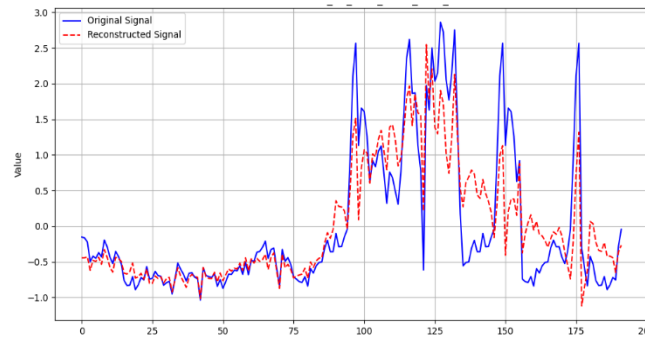


Figure 2. Signal reconstruction from the combination with MRA optimization is carried out for each cD level.

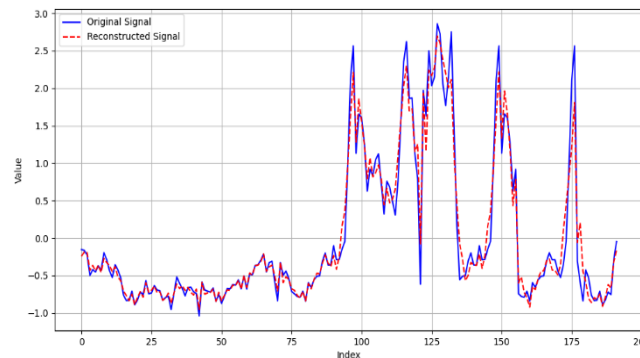


Figure 3. Signal reconstruction from a combination of threshold adaptive optimization carried out for each cD Level.

Evaluation of Algorithm Effectiveness

After the optimization algorithm model is generated, it is necessary to assess the stability of the algorithm model in finding solutions based on the combination of parameters found during optimization based on evaluation metrics. The objective function values of the evaluation metrics used to measure how well a model or solution performs in terms of achieving the desired goal are MSE, RMSE, SNR, MAPE

and Energy. Figure 4 shows the results of the model's stability in finding a solution even though the iterations increase. The graph shows a decreasing trend in error or loss over time, until it approaches a stable minimum value. This shows that the threshold optimization algorithm succeeded in finding better parameters, so that the solution became more optimal.

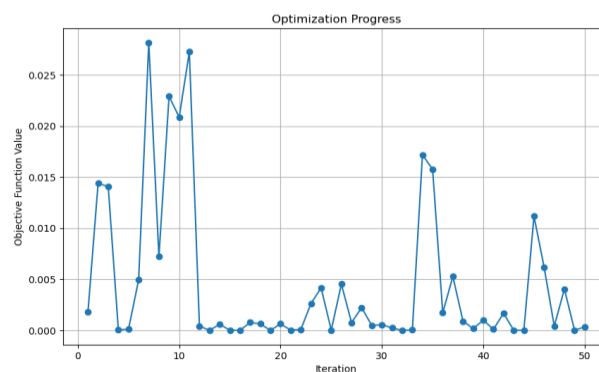


Figure 4. Graph for Evaluation of the Effectiveness of the Threshold Optimization Algorithm Model

Conclusion

Based on the identification of problems, the results of data analysis and discussion in the case study experiments of the two optimization models above, it was concluded that the application of the case study of the rupiah exchange rate against the US dollar using the Wavelet Thresholding method with MODWT transformation provides a composite value or score of various evaluation metrics MSE, RMSE, MAPE, SNR and Energy which are smaller than the MRA approach method, namely: $2.6237769439631323 < 2.8013922343357636$. So it can be concluded that for the case study of tuberculosis incident denoising which has random fluctuations from 2019-2022, the Wavelet Thresholding method is better than MRA optimization, but from several evaluation metrics the MSE value in MRA optimization has a better value, this can be a consideration for improving future optimization research.

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